2D Jump Mathematics

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Purpose

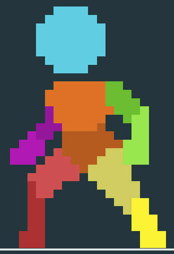
Gravity

Velocity

Jump Height

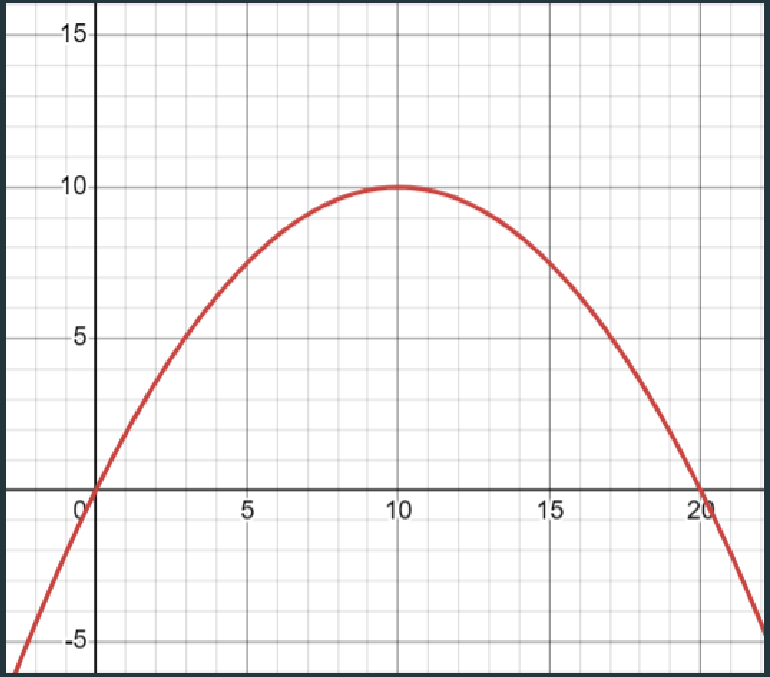
Time to Peak

By the end of this presentation, you should also have a better understanding of **quadratic equations**.



What shape does a jump follow?

That's right! A **Parabola** or **Quadratic Curve.**



More specifically

The Projectile Motion Formula

$$f(t) = \frac{1}{2}gt^2 + v_0 t + p_0$$

Where:

g = gravity
v_0 = initial velocity
p_0 = initial position

Linking the Projectile Motion Formula with the Quadratic Equation in the **General Form**

Substituting:

$$f(x) = ax^2 + bx + c$$

x $\longrightarrow t$

a $\longrightarrow \frac{1}{2}g$

b $\longrightarrow v_0$

c $\longrightarrow p_0$

$$\boxed{f(t) = \frac{1}{2}gt^2 + v_0 t + p_0}$$

General Form: $f(x) = ax^2 + bx + c$

But the Standard/Vertex Form Proves much more Useful

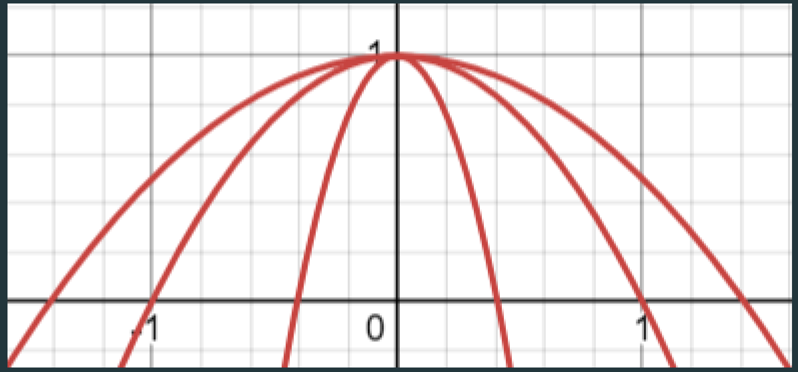
$$\boxed{f(x) = a(x^2 - \mathrm{h}) + \mathrm{k}}$$

Manipulating the general form we can deduct:

$$h = -\frac{\mathrm{b}}{2\mathrm{a}}$$

$$k = c - \frac{b^2}{4\mathrm{a}}$$

a ⟶ steepness of the curve



h ⟶ X position

k ⟶ Y position

Before we move on let's take a look at an interactive example

https://www.desmos.com/calculator/zz3vamnwzf

Linking the Projectile Motion Formula with the Quadratic Equation in the **Standard Form**

Substituting:

$$f(x) = a(x^2 - \mathrm{h}) + \mathrm{k}$$

$\mathrm{x} \longrightarrow t$

$\mathrm{a} \longrightarrow \frac{1}{2}g$

$\mathrm{h} \longrightarrow -\frac{v_0}{\mathrm{g}}$

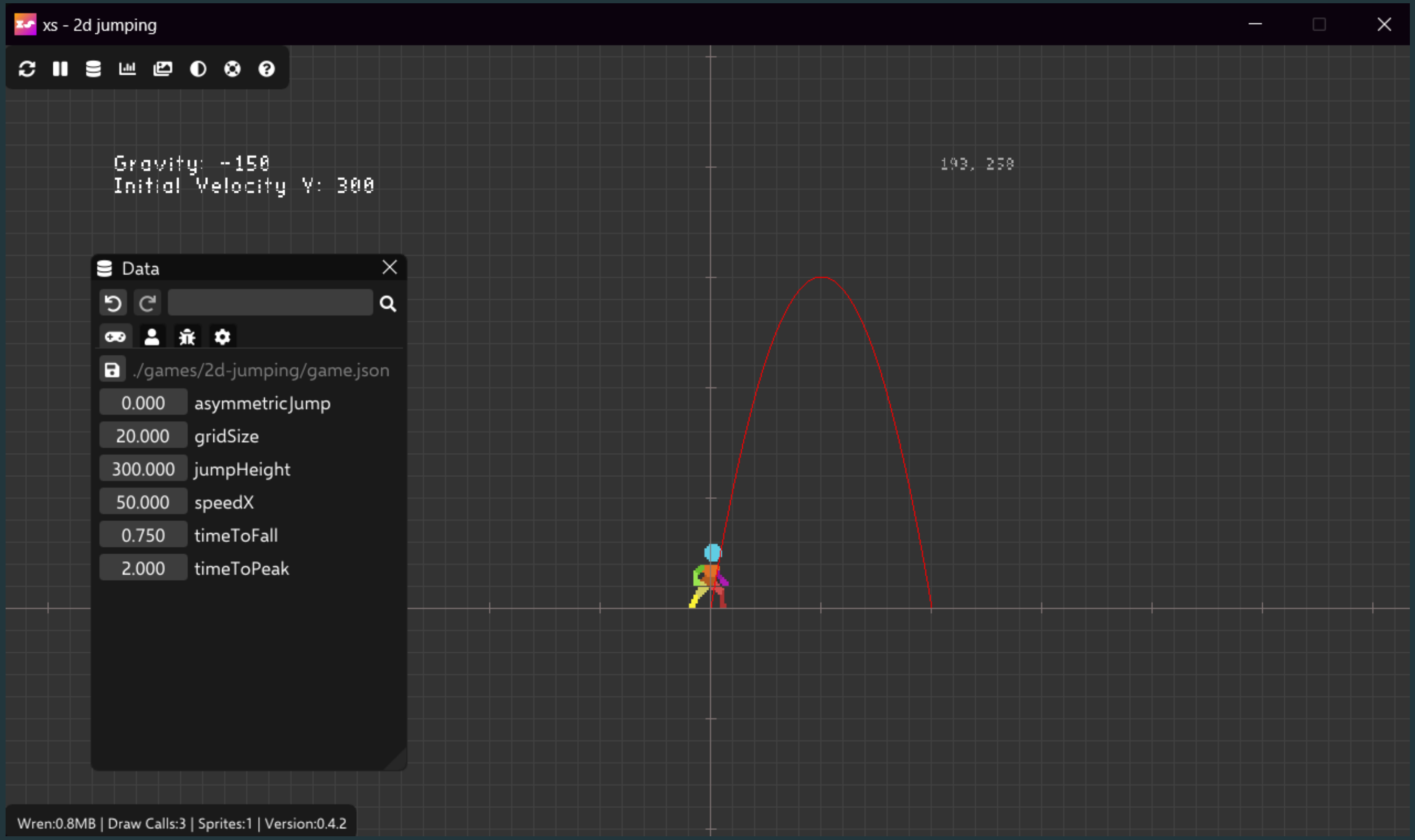
$\mathrm{k} \longrightarrow p_0 - \frac{{v_0}^2}{2\mathrm{g}}$

$$\boxed{f(t) = \frac{1}{2}g\left(\mathrm{t} + \frac{v_0}{\mathrm{g}}\right)^2 + p_0 - \frac{{v_0}^2}{2\mathrm{g}}}$$

Formula in Action

How effective will it be to have to tweak the gravity and velocity?

Let's find out by trying out my demo.



Mapping in relation to time: https://www.desmos.com/calculator/cnvbkzekud

The Intuitive Approach

Through manipulations we get:

$$v_0 = \frac{2h}{t_h}$$

$$g = -\frac{2h}{{t_h}^2}$$

$$\boxed{f(t) = \frac{1}{2}g\left(t - \left(-\frac{v_0}{g}\right)\right)^2 + p_0 - \frac{{v_0}^2}{2g}}$$

$$\underbrace{\frac{1}{2}g}_{a} \qquad \underbrace{\left(-\frac{v_0}{g}\right)}_{h} \qquad \underbrace{p_0 - \frac{{v_0}^2}{2g}}_{k}$$

h = height of jump

t_h = time to peak

Let's look at the improved version of my demo to demonstrate.

Desmos toy: https://www.desmos.com/calculator/zhugqbhbgi

Which Form to Use?

I found it easiest to use the

General Form

for actual jump **implementation**.

```
if (__inAir) {
    __posY = Game.QuadraticGeneral(dt, 1 / 2 * __gravity, __velocityY, __posY)
    __velocityY = __velocityY + __gravity * dt
}
```

But much better to use the

Standard Form

for the **visualization**.

```
var a = __gravity / (2 * __speedX * __speedX)
var h = curveXPos - __speedX * __initialVelocityY / __gravity
var k = -__initialVelocityY * __initialVelocityY / (2 * __gravity) + __groundHeight
var step = 1

var xMin = curveXPos
var xMax = xMin + __speedX * __timeToPeak * 2

Game.DrawParabolaStandard(a, h, k, color, step, xMin, xMax, __groundHeight, Num.infinity)
```

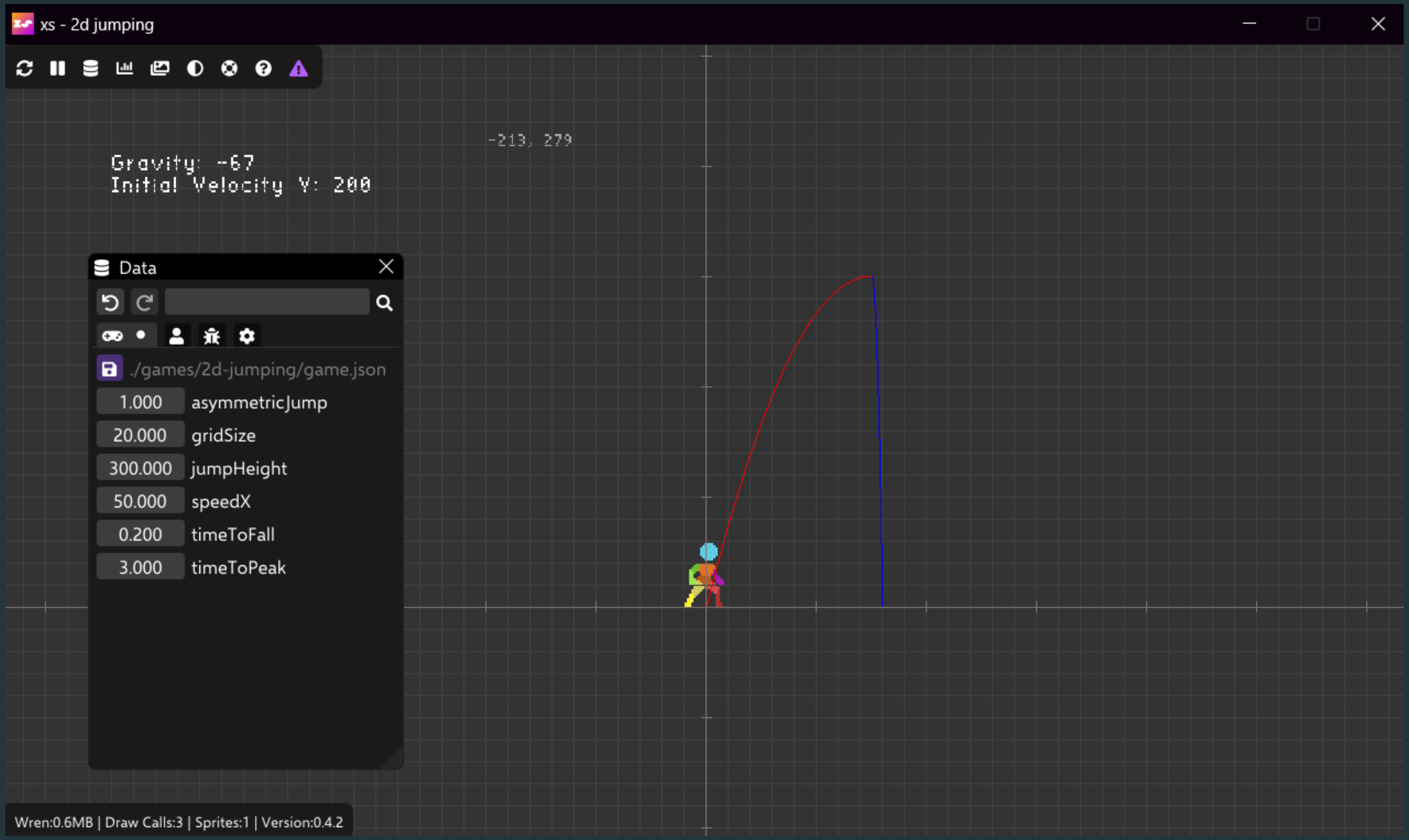
*Code uses Wren scripting language

Expansion

For this demonstration I implemented different jump/fall times.

This concept can also be expanded into composite curves:

- Different times to ascend/descend
- Double jumps



Video demo: https://youtu.be/Lo9Msm9ZIRg

Thank You for Listening!

Questions?